Analysis of Inductive WPT

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Outline

» Introduction
» Efficiency
» Power balance
» Input impedance
» Conclusion
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Introduction

Source
Transceiving coil
Receiving coil
Appliance
Transceiving coil
Receiving coil

Receiving coils
Transceiving coils
Appliance
Sources

Receiving coil
Appliance
Transceiving coils
General WPT Chain

Source -> FC -> MN -> CE
Side of source
Adaptor

Transmission medium

CE -> MN -> FC -> Appliance
Adaptor
Side of appliance
Circuit Model

a) Source + FC

Side of source

b) Appliance + FC

MN

CL

c) Appliance + FC

MN

CL

Side of appliance
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Efficiency Definition

» Main losses are caused by resistances of transceiving and receiving coils.

\[
\eta_l = \frac{P_L}{P_T} = \frac{P_L}{P_A + P_L + P_S}
\]
Efficiency Derivation (1)

\[
\eta_I = \frac{R_L X_K^2}{\left( (R_A + R_L)^2 + (X_A + X_L)^2 \right) R_S + (R_A + R_L) X_K^2}
\]

\[X_A = \omega L_A, \quad X_S = \omega L_S, \quad X_K = \omega k_I \sqrt{L_A L_S}\]

\[
\frac{X_K}{\sqrt{X_A X_S}} = k_I, \quad \frac{X_A}{R_A} = Q_A, \quad \frac{X_L}{R_A} = -Q_A', \quad \frac{X_A}{R_L} = Q_L, \quad \frac{X_S}{R_S} = Q_S
\]

\[
\eta_I = \frac{k_I^2 Q_A Q_S}{Q_L} \left( 1 + \frac{Q_A}{Q_L} \right)^2 + (Q_A - Q_A')^2 + k_I^2 Q_A Q_S \left( 1 + \frac{Q_A}{Q_L} \right) \]
Efficiency Derivation (2)

\[ \kappa = k_l \sqrt{Q_A Q_S}, \quad \rho = \frac{Q_A}{Q_L}, \quad \xi = Q_A - Q_A' \]

\[ \eta_l = \frac{\kappa^2 \rho}{(1 + \rho)^2 + \xi^2 + \kappa^2 (1 + \rho)} \]

\[ R, L, C, \omega, (X) \rightarrow k_l, Q_A, Q_A', Q_L, Q_S \rightarrow \kappa, \rho, \xi \rightarrow \]

\[ \max (\eta_l (\rho, \xi)) \rightarrow \rho = \sqrt{1 + \kappa^2}, \quad \xi = 0 \rightarrow \]

\[ \kappa, \rho, \xi \rightarrow k_l, Q_A, Q_A', Q_L, Q_S \rightarrow R, L, C, \omega, (X) \]
Maximal Efficiency, Optimal Conditions

\[ R_L = \sqrt{R_A^2 + \omega^2 k_l^2 L_A L_S \frac{R_A}{R_S}} \]

\[ X_L = -\omega L_A \]

\[ \eta_i = \frac{(k_l Q)^2}{\left(1 + \sqrt{1+(k_l Q)^2}\right)^2} \]

\[ Q = \sqrt{Q_A Q_S} \]
Maximal Efficiency

\[
\eta_1 = \frac{(k_Q)^2}{\left(1 + \sqrt{1 + (k_Q)^2}\right)^2}
\]
Max. eff., Opt. Con. – Dual Case

\[ R_L + jX_L = \frac{R'_L X'_L^2}{R'_L^2 + X'_L^2} + j\frac{R'_L^2 X'_L}{R'_L^2 + X'_L^2} \]

\[ R'_L = \sqrt{\frac{\omega^2 L_A^2}{R_A^2 + \omega^2 k^2 L_A L_S \frac{R_A}{R_S}}} + \sqrt{R_A^2 + \omega^2 k^2 L_A L_S \frac{R_A}{R_S}} \]

\[ X'_L = -\omega L_A - \frac{R_A \omega L_A}{R_S} \]
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Maximal Appliance Power, Optimal Condition

\[ \text{max}\left(P_L\left(X_M\right)\right) \]

\[ X_M = -\omega L_S \]
Power Balance (1)

\[
R_L = \sqrt{R_A^2 + \omega^2 k^2 L_A L_S \frac{R_A}{R_S}}
\]

\[
X_L = -\omega L_A
\]

\[
X_M = -\omega L_S
\]
Power Balance (2)

\[ P_A = \frac{(k_iQ)^2}{\left(1 + (k_iQ)^2 + \sqrt{1 + (k_iQ)^2}\right)^2} \left| \frac{U_s}{R_s} \right|^2 \]

\[ P_L = \frac{(k_iQ)^2 \sqrt{1 + (k_iQ)^2}}{\left(1 + (k_iQ)^2 + \sqrt{1 + (k_iQ)^2}\right)^2} \left| \frac{U_s}{R_s} \right|^2 \]

\[ P_S = \frac{\left(1 + \sqrt{1 + (k_iQ)^2}\right)^2}{\left(1 + (k_iQ)^2 + \sqrt{1 + (k_iQ)^2}\right)^2} \left| \frac{U_s}{R_s} \right|^2 \]
Power Balance (3)

\[ p_A, p_L, p_S [-] \]

\[ k_i Q [-] \]
Efficiency and Power Balance

![Graphs showing Efficiency and Power Balance](image-url)
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Input Impedance (1)

\[ Z_s = \frac{U_S}{I_S} \]
Input Impedance (2)

\[
Z_S = \frac{U_S}{I_S} = R_S + \frac{(R_A + R_L)X_K^2}{(R_A + R_L)^2 + (X_A + X_L)^2} + \\
+ j\left( X_M + X_S - \frac{(X_A + X_L)X_K^2}{(R_A + R_L)^2 + (X_A + X_L)^2} \right)
\]

By optimal conditions for efficiency and appliance power:

\[
Z_S = R_S + \frac{\omega^2 k^2 L_A L_S}{R_A + \sqrt{R_A^2 + \omega^2 k^2 L_A L_S} \frac{R_A}{R_S}}
\]
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Conclusion

» The general circuit model for WPT by electromagnetic induction was shown.
» The efficiency was mentioned.
» The power balance in the terms of normalized powers was derived.
» The input impedance was presented.